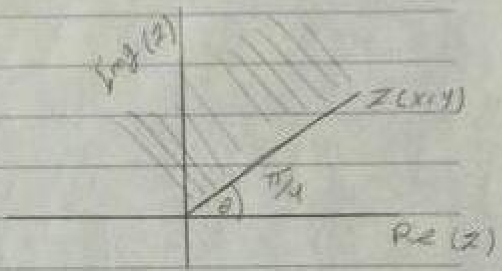
Lect 2

Graphical representation of equations in complex plane

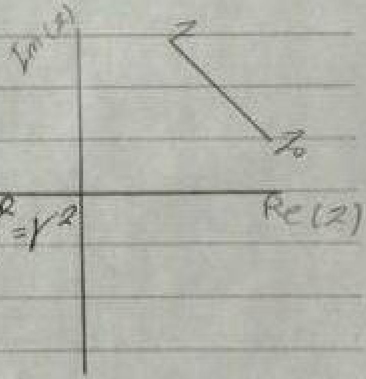
* $\text{Arg } Z \geq \pi/4$


Img(Z)
Z(x,y)
π/4
θ
Re(Z)

→ $|Z - Z_0| = r$

$Z = (x, y) \quad Z_0 = (x_0, y_0)$

$r = \sqrt{(x-x_0)^2+(y-y_0)^2} = r \rightarrow (x-x_0)^2+(y-y_0)^2 = r^2$



= Harmonic function =

$f = f(x,y)$ $\Delta f = 0$ $\therefore f \rightarrow$ Harmonic function

"laPlace equation" ↲

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ f is Harmonic $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

if function $f(x,y)$ is analytic "entire"

$\ast \rightarrow f = u + iv \rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \rightarrow ①$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \rightarrow ②$

analytic f لو]

Dif ① with relative to x

$\hookrightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y}$.

Dif ② with relative to y

$\hookrightarrow \frac{\partial^2 u}{\partial y^2} = \frac{-\partial^2 v}{\partial x \partial y}$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ ← بالجمع

if f is analytic $\therefore$ u "Re" → harmonic

[v "Img" → harmonic

but the inverse isn't true

$\ast$ u → harmonic

$\ast$ v → harmonic $\xrightarrow[\text{الضروري}]{\text{ليس من}}$ f analytic

ex: $f = y + i(x+y)$

$\hookrightarrow u = y$ "harmonic" لان تفاضلها الثاني يعطي Zero

$\hookrightarrow v = (x+y)$ "harmonic"

تفاضلها مرتين بالنسبة لـ x ، y = Zero

u and v don't satisfy C.R. Condition

f isn't analytic

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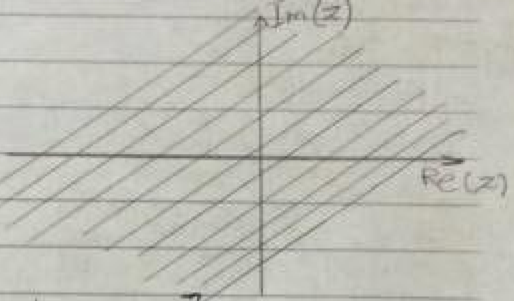
chapter 3

elementary complex functions and their properties

* properties of the function
* single value - Multi-value
* contineous
* periodic
* bounded

1) the exponential function

$w = f(z) = e^{z}$



1) domain = Z
2) contineous function
3) analytic

$\therefore e^{z} = e^{x+iy} = e^{x}[\cos y + i \sin y]$

$= e^{x}\cos y + i e^{x}\sin y \quad \therefore u = e^{x}\cos y \quad v = e^{x}\sin y$

$\frac{\partial u}{\partial x} = e^{x}\cos y \quad \frac{\partial v}{\partial y} = e^{x}\cos y \quad \therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} \rightarrow ①$

$\frac{\partial u}{\partial y} = -e^{x}\sin y \quad \frac{\partial v}{\partial x} = e^{x}\sin y \quad \therefore \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \rightarrow ②$

from ① & ② $\therefore e^{z}$ is analytic "entire"

4) periodic $\rightarrow w = e^{z} = e^{x+iy} = e^{x}(\cos y + i \sin y)$

∴ القيم لا تتغير $2n\pi + y = y$

$\therefore e^{z}$ is periodic function

$w = e^{x} + i(y + 2n\pi) \quad w = e^{x+iy} + i(2n\pi) \quad w = e^{z+2n\pi i} \rightarrow$ period $2\pi i$

5) single value "e^{z}" لأن قيمة الدالة لا تتغير

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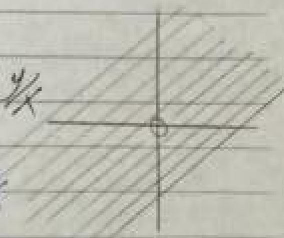
* 1) the logarithmic function *

$w = f(z) = \ln(z) = \log(z) = \ln(re^{j\theta}) = \ln r + i\theta$

1) Domain $\mathbb{C} - \{0\}$

2) Continuous $\mathbb{C} - \{0\}$ $= \ln\sqrt{x^2+y^2} + i\tan^{-1} y/x$



3) analytic $w = \ln z = \ln\sqrt{x^2+y^2} + i\tan^{-1} y/x$

$U = \ln\sqrt{x^2+y^2}$ & $V = \tan^{-1} y/x$

$\frac{\partial U}{\partial x}$ & $\frac{\partial V}{\partial y}$ $\mathbb{C} - \{0\}$

4) Periodic → $w = \ln z$ Not Periodic

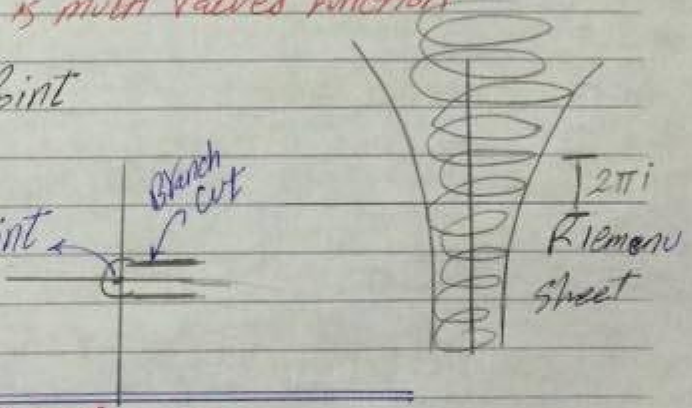
5) single valued – multi valued

$w = \ln z = \ln re^{i(\theta+2\pi n)} = \ln r + i\theta = \ln r + i(\theta + 2\pi n)$

logarithmic function is multi valued function

(0,0) is the Branch Point

الدالة عندها [illegible] تسمى Branch Point [illegible]



* the trigonometric function *

$w = \cos z \Rightarrow \mathbb{C}$

a) Domain $\mathbb{C}$ b) continuous function ✓

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c) analytic → $w = \cos z$ → $w = \cos(x+iy)$

$\therefore w = \cos x \cos iy - \sin x \sin iy$

$\therefore$ ~~$\cos iy = i \sinh y$~~ $\boxed{\begin{aligned} \sin(iy) &= i\sinh y \\ \cos iy &= \cosh y \end{aligned}}$

$w = \cos x \cosh y - i \sin x \sinh y$

$u = \cos x \cosh y$
$v = -\sin x \sinh y$ → $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

then the function is analytic

4) periodic → yes and the period 2π

5) single - multi → yes is single

3) the inverse trigonometric fn

Note

any fn has rathenal power less than one or logairthem then it ~~sould~~ should has a branch points and branch cut

$\cos y = \frac{e^{iy} + e^{-iy}}{2}$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\cos iy = \frac{e^{i(iy)} + e^{-i(iy)}}{2} = \frac{e^{y} + e^{-y}}{2}$

$\boxed{\cos iy = \cosh y}$

$\sin y = \frac{e^{iy} - e^{-iy}}{2i}$

$\sin iy = \frac{e^{i(iy)} - e^{-i(iy)}}{2i} = \frac{e^{-y} - e^{y}}{2i} \times \frac{i}{i} = \frac{ie^{y} - ie^{-y}}{2}$

$\therefore \boxed{\sin iy = i \sinh y}$

ex: $w = \sin^{-1} z$

→ $\sin w = z$ → $\frac{e^{iw} - e^{-iw}}{2i} = z$ $\therefore e^{iw} - e^{-iw} = 2iz$

multiply e^{iw} → $e^{i2w} - 1 = 2iz\, e^{iw}$ $\therefore e^{2wi} - (2iz)\, e^{iw} - 1 = 0$

$\therefore x = e^{wi} = \frac{2iz \pm \sqrt{-4z^2 + 4}}{2} = \boxed{zi \pm \sqrt{-z^2 + 1}} \rightarrow x$

$\therefore x^2 - (2iz)x - 1 = 0$ $\quad wi = \ln\left(iz \pm \sqrt{1 - z^2}\right)$

$w = \frac{1}{i} \ln\left(iz \pm \sqrt{1 - z^2}\right)$

$\therefore \boxed{\sin^{-1} z = \frac{1}{i} \ln\left(iz \pm \sqrt{1 - z^2}\right)}$

No periodic
Multiply
Continous

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